A Computer Program for Calculating Blood Acid-Base Parameters on an Olivetti "Programma 101" Desk Computer

Jørgen Hardt

A computer program is presented for a desk calculator, to be used in calculating \( P_{CO_2} \), standard bicarbonate, and base excess. The computer simulates the readings from the Siggaard-Andersen nomogram [Scand. J. Clin. Lab. Invest. 14, 598 (1962)].

Additional Keyphrases  
acid–base calculations  
base excess  
\( P_{CO_2} \)  
Siggaard-Andersen nomogram  
Astrup method

The Astrup method for determining acid–base parameters involves three pH measurements, one on the original blood sample (pH \( a \)) and two more (pH 1 and pH 2) after equilibration of the blood with two standard CO\(_2\) tensions (\( P_{CO_2} \) and \( P_{CO_2} \)). By entering these data in the Siggaard-Andersen nomogram (1), the acid–base parameters are estimated. Used routinely, this nomogram technique is tedious work. For that reason, a computer program has already been published (2), but this does not solve the problem for smaller laboratories that have no computer facilities. I have therefore constructed a program for a desk calculator that, because it is relatively inexpensive, should be available for every laboratory.

Materials and Methods

The "Programma 101" (Olivetti–Underwood) desk calculator, programmed with magnetic cards is all the hardware needed.

The computer program is worked out on the basis of the acid–base nomogram proposed by Siggaard-Andersen. In this nomogram, which is a pH/log \( P_{CO_2} \) system of coordinates, are plotted the coordinates (pH 1, \( P_{CO_2} \)) and (pH 2, \( P_{CO_2} \), (see Figure 1). The straight line through these points constitutes a standard curve, which for every pH value gives the corresponding \( P_{CO_2} \), of the blood. The \( P_{CO_2} \), corresponding to pH \( a \) is the actual carbon dioxide tension (\( P_{CO_2} \), \( a \)) of the blood. Where the standard curve intersects the base excess scale, the base excess (BE)\(^1\) is read, where it intersects the plasma bicarbonate scale the standard bicarbonate is read.

If one is to construct a computer program, these nomogram estimates must be expressed mathematically. Referring to the right-angled triangle in Figure 1 we have:

\[
\begin{align*}
\text{pH 1} - \text{pH a} &= \ln P_{CO_2,1} - \ln P_{CO_2, a} \\
\text{pH 1} - \text{pH 2} &= \ln P_{CO_2,1} - \ln P_{CO_2, 2} \text{ so that}
\end{align*}
\]

\[
\ln P_{CO_2, a} = \ln P_{CO_2,1} + \frac{\text{pH 1} - \text{pH a}}{\text{pH 1} - \text{pH 2}} \times \ln P_{CO_2, 2}
\]

(\( \ln \) = the natural logarithm).

With this computer, for technical reasons, prefer to compute \( \frac{P_{CO_2, a}}{10} \):

\[
\ln \frac{P_{CO_2, a}}{10} = \ln \frac{P_{CO_2,1}}{10} + \frac{\text{pH 1} - \text{pH a}}{\text{pH 1} - \text{pH 2}} \times \ln P_{CO_2, 2}
\]

\(^1\)Nonstandard abbreviations used: BE, base excess; and barometric pressure (given here in mm of mercury; the S equivalent is 1 mm Hg \( \approx 133 \) N/m\(^2\) \( \approx 133 \) Pa).
where \( \ln \left( \frac{P_{CO_2}}{P_{CO_2}} \right) \) for all practical purposes is constant, although small variations do occur with the barometric pressure. On the other hand, \( \ln \left( \frac{P_{CO_2}}{1/10} \right) \) has to be corrected for deviations in barometric pressure (\( \beta P \)). An expression can be found, that gives \( \ln \left( \frac{P_{CO_2}}{1/10} \right) \) as a function of barometric pressure.

We get the \( \text{CO}_2 \) gas standards from a gas-mixing apparatus (GMA-1 Radiometer, Copenhagen), which gives \( P_{CO_2} = 25.6 \) mm Hg and \( P_{CO_2} = 51.4 \) mm Hg at \( \beta P = 760 \) mm Hg. This means that

\[
\ln \frac{P_{CO_2}}{1/10} = \beta P - 84
\]

\[\text{and} \quad \ln \frac{P_{CO_2}}{P_{CO_2}} = 0.697\]

The equation calculated by the computer is thus

\[
\ln \frac{P_{CO_2}}{1/10} = \frac{\beta P - 84}{720} + \frac{\ln P_{CO_2}}{\ln P_{CO_2}} \times 0.697 \quad (1)
\]

The antilogarithm is afterwards calculated by means of the (Taylor) series expansion of the exponential function:

\[
e^x = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \ldots + \frac{x^n}{n!}
\]

To computed the standard bicarbonate we use the Henderson-Hasselbalch equation:

\[
\text{pH} = 6.10 + \log \frac{[\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3]} \quad \text{so that}
\]

\[
\log [\text{HCO}_3^-] = \text{pH} - 6.021 \quad \text{and}
\]

\[
\ln \frac{[\text{HCO}_3^-]}{10} = 2.303 (\text{pH} - 7.021) \quad (2)
\]

(\( \log = \text{the common logarithm} \)), \( \text{pH} \) is computed by means of the standard curve (Figure 1). Referring to the triangles, we have

\[
\frac{\text{pH} - \text{pH}}{\text{pH} - \text{pH}} = \ln 40 - \ln P_{CO_2} - \ln P_{CO_2} - \ln P_{CO_2} - \ln P_{CO_2}
\]

The right side of the equation can be calculated as a constant (K) so that

\[
\text{pH} = \text{pH} - K(\text{pH} - \text{pH})
\]

which, put into Equation 2, gives \( \ln ([\text{HCO}_3^-]/10) \).

\( \text{BE} \) is computed by means of \( \text{pH} \), \( \text{pH} \), and the constants \( P_{CO_2} \) and \( P_{CO_2} \). Consequently, \( \text{BE} \) can be expressed as a mathematical function of \( \text{pH} \) and \( \text{pH} \): \( \text{BE} = F(\text{pH}, \text{pH}) \). With enough \( \text{pH} \) values (\( \text{pH} \), \( \text{pH} \)) and their corresponding \( \text{BE} \) values, it is possible to find an approximation for \( F(\text{pH}, \text{pH}) \) using the first terms of Taylor's formula:

\[
F(\text{pH}, \text{pH}) = F(''\text{pH},'' ''\text{pH}'') + \frac{\partial F}{\partial \text{pH}} (\text{pH} - ''\text{pH}'') + \frac{\partial F}{\partial \text{pH}} (\text{pH} - ''\text{pH}'') (\text{pH} - ''\text{pH}'') (4)
\]

The coordinate ("\( \text{pH} 1 \), "\( \text{pH} 2 \)) is a fixed point, \( \text{pH} 2 \) is the partial derivative with respect to \( \text{pH} 1 \), and \( \text{pH} 2 \) is the partial derivative with respect to \( \text{pH} 2 \).

The partial derivatives are determined by drawing two graphs, one showing \( \text{BE} \) as a function of \( \text{pH} 1 \) with \( \text{pH} 2 \) kept constant, and the other showing \( \text{BE} \) as a function of \( \text{pH} 2 \) with \( \text{pH} 1 \) kept constant. \( \frac{\partial F}{\partial \text{pH} 1} \) is then the tangent gradient at the point ("\( \text{pH} 1 \), "\( \text{pH} 2 \)") on the first graph, and \( \frac{\partial F}{\partial \text{pH} 2} \) is the tangent gradient at the same point on the second graph. If only a few terms of the Taylor formula are used, the approximation for \( F(\text{pH}, \text{pH}) \) works well only in a limited area [around ("\( \text{pH} 1 \), "\( \text{pH} 2 \)")]. To cover all the \( \text{pH} \) values in the nomogram it therefore is necessary to find three equations for \( \text{BE} \), each working in its own \( \text{pH} \)-area. With our gas mixtures, the three equations are:

\[
\begin{align*}
F_1(\text{pH}, \text{pH}) & = 36 \text{ pH} + 3.3 \text{ pH} - 299 \quad (A) \\
F_2(\text{pH}, \text{pH}) & = 20 \text{ pH} + 40 \text{ pH} - 444 \quad (B) \\
F_3(\text{pH}, \text{pH}) & = 32 \text{ pH} + 43 \text{ pH} - 566 \quad (C)
\end{align*}
\]

On the basis of the mean (\( \text{pH}_M \)) of \( \text{pH} 1 \) and \( \text{pH} 2 \), the computer chooses the appropriate equation:

\[
\begin{align*}
\text{For } \text{pH}_M & \leq 7.30, \text{ formula A is used} \\
7.30 < \text{pH}_M & \leq 7.46, \text{ formula B is used} \\
\text{pH}_M > 7.46, \text{ formula C is used}
\end{align*}
\]
Description of the Programs

The computer program that calculates $P_{CO_2}$ and standard bicarbonate is used as follows:

The barometric pressure is fed into the computer and the first term ($BP - 84/720$) in Equation 1 is computed and stored. pH 1 and pH 2 are entered next and pH from Equation 3 is calculated and stored. Actual pH is now entered, the second term in Equation 1 is calculated, the first term is added, and the computation of the antilogarithm of $\ln(1/10) P_{CO_2}$ begins. The calculations now go into a loop, that calculates and adds term after term in the expression for $e^x$, until the value of the last term is less than 0.001. In this computation it is an advantage to use $\ln (1/10) P_{CO_2}$, $a$ instead of $\ln P_{CO_2}$, $a$, because fewer terms have to be calculated before the value of the last term is less than 0.001. This saves time and more terms would not give smaller errors. After the antilogarithm calculation, $(1/10) P_{CO_2}$, is multiplied by 10 and the resulting $P_{CO_2}$ in blood is printed out. A conditional jump instruction follows and the calculation of the standard bicarbonate begins.

The pH calculated above is brought from storage, and $ln (1/10)[HCO_3^-]$ is computed according to Equation 2. A jump instruction starts the antilogarithm calculation in the previously mentioned loop. The $(1/10)[HCO_3^-]$ is multiplied by 10 and the result is read out. A conditional jump instruction reads the computer for new pH values.

The program for BE works as follows: The two pH values (pH 1 and pH 2) are entered, and the difference is calculated. If this difference is greater than 0.24 or less than 0.16 the computer stops. This is a test to ensure that incorrect results owing to obvious analytical error are omitted.

If the pH values are accepted, their mean value is calculated and the computer chooses the appropriate equation from the program. BE is calculated and the result printed out.

Results

The calculation of the $P_{CO_2}$, $a$ and the standard bicarbonate are exact simulations of the nomogram, as is reflected by the regression lines. The regression line between the computed $P_{CO_2}$, $a$ values [$P_{CO_2}, (C)$] and the nomogram determined $P_{CO_2}, a$ values [$P_{CO_2}, (N)$] is: $P_{CO_2}, (N) = 0.25 + 0.98 P_{CO_2}, (C); r = 0.999$ (27 samples). The regression line between the computed standard bicarbonate values [HCO$_3^-$ (C)] and the nomogram determined standard bicarbonate values [HCO$_3^-$ (N)] is: HCO$_3^-$ (N) = 1.42 + 0.94 HCO$_3^-$ (C), $r = 0.999$ (27 samples).

The expression for BE is empiric and the deviation from the nomogram has been carefully ex-
amined for a series of (pH 1, pH 2) values. Table 1 gives a selection of our calculations, including the maximal discrepancies between the nomogram readings and the computer calculations. The smallest discrepancies are found in the middle of the three function areas.

Our result indicates that the nomogram readings can be changed to a calculation by a desk computer. We have used the computer for the acid–base calculations since December 1, 1971, with good results. The computer is easy to operate, it is difficult to make mistakes, and control has been facilitated, because computational errors need not be looked for; the only thing to control is that accurate values have been entered, which appears from the computer printouts.

References


Table 1. Comparison between Base Excess (BE) Calculated by the Computer (\( BE_\text{C} \)) and BE Estimated from the Nomogram (\( BE_\text{N} \))

| pH 1 | pH 2  | \( BE_\text{C} \) | \( BE_\text{N} \) | \(| \Delta BE | \) |
|------|-------|------------------|------------------|----------|
| 7.15 | 6.98  | -18.7            | -18.6            | 0.1      |
| 7.17 | 7.00  | -18.0            | -17.8            | 0.2      |
| 7.20 | 7.01  | -16.6            | -16.7            | 0.1      |
| 7.23 | 7.04  | -15.5            | -15.5            | 0.0      |
| 7.27 | 7.09  | -14.0            | -13.9            | 0.1      |
| 7.31 | 7.14  | -12.0            | -12.3            | 0.3      |
| 7.35 | 7.17  | -10.0            | -10.7            | 0.7      |
| 7.39 | 7.20  | -8.1             | -9.2             | 1.1      |
| 7.42 | 7.25  | -6.0             | -5.6             | 0.4      |
| 7.46 | 7.27  | -4.9             | -4.0             | 0.0      |
| 7.50 | 7.32  | -1.0             | -1.2             | 0.2      |
| 7.54 | 7.35  | 1.0              | 0.8              | 0.2      |
| 7.57 | 7.40  | 4.6              | 4.5              | 0.1      |
| 7.60 | 7.48  | 6.2              | 6.3              | 0.1      |
| 7.64 | 7.45  | 8.8              | 8.8              | 0.0      |
| 7.68 | 7.49  | 11.0             | 11.8             | 0.8      |
| 7.72 | 7.53  | 15.0             | 14.8             | 0.2      |
| 7.75 | 7.57  | 16.4             | 17.5             | 1.1      |
| 7.77 | 7.59  | 18.7             | 19.0             | 0.3      |